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කල්විප් பொதுத் தராதரம் பத்திர (உயர் தர)ப்.

General Certificate of Education (Adv. Level) Examination, August

2021

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கணிதம் I

Combined Mathematics I

03 Hours

### Instructions

- This question paper consists of two parts; Part A (questions 1 -10) and part B (questions 11- 17).

#### Part - A

- Answer all questions. Answers should be written in the space provided on the question paper. If additional space is needed, you may use additional answer sheets.

#### Part - B

- Answer only 5 questions.
- After the allocated time hand over the paper to the supervisor with both parts attached together.
- Only part B of the paper is allowed to be taken out of the examination hall.

Combined Maths		
Part	Question	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Combined Maths

Final Marks

Gayan M. Perera

B.Sc. (Hons) Maths Special

M.Sc. (Financial Mathematics)

**Part A.**

01. Expand  $(2x - 3y + 4z)^2$

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02. Expand  $\left(\frac{1}{2x} - \frac{1}{3y}\right)^2$

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03. If  $\frac{x^2-1}{x} = 4$  show that  $\frac{x^6-1}{x^3} = 76$

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04. If  $a + b - 3 = 0$  then find the value of  $a^3 + b^3 + 9ab - 26$

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05. Factorize  $x^4 + 5x^2y^2 + 4y^4$

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06. Factorize  $(x^2 - 3x)^2 - 38(x^2 - 3x) - 80$

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07. Solve  $(x + \frac{6}{x})^2 - 12(x + \frac{6}{x}) = -35$

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08. Evaluate  $\frac{1}{\log_{xy}xyz} + \frac{1}{\log_{yz}xyz} + \frac{1}{\log_{zx}xyz}$

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09. If  $a : b = c : d$  show that  $(2a + 3b) : (2c + 3d) = (2a - 3b) : (2c - 3d)$

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10. If  $\log(x + y) = \log x - \log y$  show that  $x(1 - y) = y^2$

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## Part B

11. (i) Show that  $\frac{\log 3\sqrt{3} + \log 2\sqrt{2} - \log 5\sqrt{5}}{\log 1.2} = \frac{3}{2}$

(ii) Prove that  $\log_a b = \frac{\log_c b}{\log_c a}$

(iii) Hence show that

(a)  $\log_{ab} x = \frac{\log_a x}{1 + \log_a b}$

(b) (i)  $\frac{1}{\log_a P} + \frac{1}{\log_b P} + \frac{1}{\log_c P} = \frac{1}{\log_x P}$  where  $x = abc$

(ii) By choosing suitable values for  $a, b, c$  and  $P$  evaluate the

following expression.  $\frac{1}{\log_2 27,000} + \frac{1}{\log_3 27,000} + \frac{1}{\log_5 27,000}$

12. (i) If  $x^2 + y^2 = 7xy$ , then show that  $\log \left\{ \frac{x+y}{3} \right\} = \frac{1}{2} \{ \log x + \log y \}$

(ii) If  $\log a = n - 1$ ,  $\log b = n$  and  $\log c = n + 1$ , then prove the following

(a)  $\log (abc)^{\frac{1}{3}} = n$

(b)  $\log(ac)^{\left(\frac{1}{2}\right)} = n$

(c)  $ac = b^2$

(iii) Show that  $\log_a b = \frac{1}{\log_b a}$ . Hence prove that,

(a)  $\log_{ab} x = \frac{1}{\log_x a + \log_x b}$

(b)  $\frac{1}{\log_2 2019} + \frac{1}{\log_4 2019} + \frac{1}{\log_8 2019} \dots \dots \dots \frac{1}{\log_{100} 2019} = \frac{1}{\log_{100} 2019}$

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13.(a) If  $\log a = n - 1$ ,  $\log b = n$  and  $\log c = n + 1$ , then prove the following

(i)  $\log \frac{c}{a} + \log \frac{b}{a} + \log \frac{a}{c} = 4$

(ii)  $(\log b)^2 - (\log a)(\log c) = 1$

(iii)  $(\log c)^2 = (\log a)^2 + \log b^4$

(b) If  $x^2 + y^2 = 7xy$ , show that

(i)  $2 \log(x + y) = 2 \log 3 + \log x + \log y$

(c) Show that  $\frac{\log 27 + \log 8 - \log 125}{\log 6 - \log 5} = 3$

(d) Show that  $\log_e(e^2 - e) = \log_e(e - 1) + 1$

14. (i) Solve  $\frac{2x+1}{1-x} = \frac{3-2x}{x+1}$

(ii) If I add 13 to my age then I get my uncle's age. Three years before my age was exactly a half of my uncle's age. Find my uncle's age.

(iv) Solve the following set of simultaneous equations.

$$3x + 5y - 14 = 0$$

$$2x + 3y + 2z = 13$$

$$3x - 2y - z = 25$$

(v) Solve the following equations use a substitution where necessary.

(i)  $\left(\frac{x}{x+1}\right)^2 + 5\left(\frac{x}{x+1}\right) + 6 = 0$

(ii)  $\sqrt{\frac{x}{x-1}} + \sqrt{\frac{x-1}{x}} = 2\frac{1}{6}$

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15. (i) Simplify  $\frac{a^3+b^3}{a(a^2-b^2)} \times \frac{a+b}{a-b} \times \frac{a^2-ab}{(a+b)^2}$
- (ii) If  $p = \frac{1-t}{1+t}$  then express  $\frac{1-p^2}{1+p^2}$  in terms of  $t$
- (iii) If  $x = \frac{1+t}{1-t}$  and  $y = \frac{1-t}{1+t}$  then express  $\frac{x-y}{1+xy}$  in terms of  $t$
- (iv) Solve  $x^{\frac{1}{2}} - x^{-\frac{1}{2}} = \frac{3}{2}$
- (v) If  $W = x^{a-b}$  and  $V = x^{a+2b}$  then express  $\frac{W}{V}$  and  $\frac{W^2}{V}$  in terms of  $x, a$  and  $b$
- (vi) Simplify  $\{xy\}^{a-2b} \times \left\{\frac{x^2}{y}\right\}^{2a+b} \times \{x^{-1}y\}^{a-b}$
16. (i) Expand and simplify,
- (a)  $\left(a + \frac{1}{2a}\right)^3$       (b)  $(x + y - z)^2$
- (ii) If  $x + y = 1 + xy$ , then show that  $x^3 + y^3 = 1 + (xy)^3$
- (iii) If  $x + y - 7 = 0$  and  $xy = 12$  then find the value of
- $$x^3 + y^3 + 4xy(x + y)$$
- (iv) Show that  $\frac{1}{(x-y)(x+z)} \left\{ \frac{(x+y)^3 - (y+z)^3}{x-z} - \frac{(x+y)^3 + (y+z)^3}{x+2y+z} \right\} = 2$



17. (i) Simplify

$$(a) \left\{ \frac{a}{b} \right\}^{(x+2y)} \times \{ab\}^{x-y} \times \{b^2\}^{\frac{x+y}{2}} \quad (b) \left\{ \frac{x^a}{x^b} \right\}^c \times \left\{ \frac{x^b}{x^c} \right\}^a \times \left\{ \frac{x^c}{x^a} \right\}^b$$

(ii) Solve

$$(a) (3^x)^2 - 12 \times 3^x + 27 = 0 \quad (b) 4 \log_{16} x - 1 = \log_x 4$$

$$(c) 3 \left[ (x+7)^{\frac{1}{2}} + (x+7)^{-\frac{1}{2}} \right] = 10$$

$$(d) x + y = 4$$

$$x^2 - y = 8$$

$$(iii) \text{ If } x = \frac{6ab}{a+b} \text{ find the value of } \frac{x+3a}{x-3a} + \frac{x+3b}{x-3b}$$